

General Displaced $SU(1,1)$ number states - revisited

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Abstract

The most general displaced number states, based on the bosonic and an irreducible representation (IREP) of the Lie algebra symmetry of $su(1,1)$ and associated to the Calogero-Sutherland model are introduced. Here, we utilize the Barut-Girardello displacement operator instead of the Klauder-Perelomov counterpart, to construct new kind of the displaced number states which can be classified in nonlinear coherent states regime, too, with special nonlinearity functions. They depend on two parameters, and can be converted into the well known Barut-Girardello coherent and number states respectively, depending on which of the parameters equal to zero. A discussion of the statistical properties of these states is included. Significant are their squeezing properties and anti bunching effects which can be raised by increasing the energy quantum number. Depending on the particular choice of the parameters of the above scenario, we are able to determine the status of compliance with flexible statistics. Major parts of the issue is spent on something that these states, in fact, should be considered as new kind of photon-added coherent states, too. Which can be reproduced through an iterated action of a creation operator on new *nonlinear Barut-Girardello coherent states*. Where the latter carry, also, outstanding statistical features.

keywords: Photon-added coherent state, Sub-Poissonian, Squeezing, Anti-bunching, Entanglement.

1 Introduction

In recent years, many authors have investigated new quantum states of the electromagnetic field such as coherent states [1- 12] which provide us with a link between quantum and classical mechanics and nowadays pervade many branches of physics including quantum electrodynamics, solid-state physics, and nuclear and atomic physics, from both theoretical and experimental viewpoints. In addition to CSs, squeezed states (SSs) are becoming increasingly important. These are the non-classical states of the electromagnetic field in which certain observables exhibit fluctuations less than in the vacuum state [13]. These states

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are important because they can achieve lower quantum noise than the zero-point fluctuations of the vacuum or coherent states. Over the last four decades there have been several experimental demonstrations of nonclassical effects, such as the photon anti-bunching [14], sub-Poissonian statistics [15], and squeezing [16].

Besides the above developments, on the one hand, special attention has also been carried out to investigate the properties of the displaced number states (DNSs),

$$\begin{aligned} DNSs &:\equiv \mathcal{D}_{KP}(z)|m, \lambda\rangle \\ \mathcal{D}_{KP}(z) &:\equiv \textit{Klauder - Perelomov type of Displacement Operator} \\ |m, \lambda\rangle &= \textit{Arbitrary Fock States} \end{aligned}$$

which were given by Plebanski in 1954 [17] and more analyzed by Boiteux in 1973 [18] under the general name semi coherent states overall differences than what was presented previously in [19] (the reader can find comprehensive information about DNSs in [20, 21]). It has been shown that such states have interesting and unusual physical properties. Since DNSs are obtained from a number states by adding a non-zero value to field amplitude, the state becomes phase dependent because of the phase of the displacement. The fact that these phase dependent makes it interesting to study their phase properties [22]. Also, the construction of DNSs is studied [23] corresponding with a free particle moving on sphere. There the structure of the Klauder- Perelomov coherent states associated to the $su(1,1)$ algebra is considered.

On the other hand, recently, significant progress was achieved in development of the nonlinear coherent states(NLCSs) or f-CSs, $|z, f\rangle$, which are described by a specified non-linearity function $f(\hat{N})$ [24] and linked to nonlinear or deformed algebras rather than the Lie algebras. NLCSs are defined as the right-hand eigenstates of the non-Hermitian and deformed annihilation operator $f(\hat{N})\hat{a}$, i.e.

$$f(\hat{N})\hat{a}|z, f\rangle = z|z, f\rangle$$

,or by applying the generalized displacement operator on the ground state:

$$|z, f\rangle = e^{\overline{f(\hat{N}-1)}^z \hat{a}^\dagger} |0\rangle,$$

where \hat{a} and \hat{a}^\dagger are the annihilation and creation operators of the harmonic oscillator, $f(\hat{N})$ is an operator-valued function of the number operator $\hat{N} = \hat{a}^\dagger \hat{a}$. Obviously, $|z, f\rangle$ becomes the canonical coherent state when $f(\hat{N}) = 1$. In fact the nature of the nonlinearity depends on the choice of the function $f(\hat{N})$ [25, 26]. These states may appear as stationary states of the center-of mass motion of a trapped ion [24], and exhibit nonclassical features such as quadrature squeezing, sub-Poissonian statistics, anti-bunching as well as self-splitting effects [27, 28, 29, 30].

Thus this work is devoted to analysis and pay more attention on the possibility of construction of DNSs which can be raised systematically, in an algebraic way, with the help of the Barut-Girardello type of displacement operators, $\mathcal{D}_{BG}(\mathfrak{z})$, instead of the Klauder- Perelomov counterpart, acting on arbitrary Fock states $|m, \lambda\rangle$. As will be shown, they illustrate more pronounced and controllable statistical as well as nonclassical properties than the well

known DNSs and this is main reason to study them, carefully. In order to justify the effectiveness of this technique, wide range of non-classical quantum states associated to Lie group $SU(1, 1)$ and corresponding with the Calogero-Sutherland model will be constructed. Because of the two-particle Calogero-Sutherland model has attracted considerable interests [31, 32, 33] and enjoys the $su(1, 1)$ dynamic symmetry [34, 35], too. It is of great interest in quantum optics because it can characterize many kinds of quantum optical systems [7, 8, 36]. In particular, the bosonic realization of $su(1, 1)$ describes the degenerate and non-degenerate parametric amplifiers [37]. We will prove that they are actually new kinds of *photon-added type of Barut-Girardello coherent states* (PABGCSs), $||\mathfrak{z}, m\rangle_\lambda$, because of these states are emerged through an iterated action of a creation operator, J_+^λ , on *nonlinear Barut-Girardello coherent states* (NBGCSs), $|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$. While the latter are obtained through an action of m -deformed Barut-Girardello displacement operator, $\mathcal{D}_{BG}^m(\mathfrak{z})$, on vacuum state, $|0, \lambda\rangle$. The reason for using the word nonlinear is that, $|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$ include nonclassical features such as squeezing, anti-bunching effects and sub-Poissonian statistics. The outcome of the above discussion can be summarized in the following commutative diagram

$$\begin{array}{ccc}
& \mathcal{D}_{BG}^m(\mathfrak{z}) & \\
|0, \lambda\rangle & \Rightarrow & |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda \\
(J_+^\lambda)^m \Downarrow & & \Downarrow (J_+^\lambda)^m \\
& \mathcal{D}_{BG}(\mathfrak{z}) & \\
|m, \lambda\rangle & \Rightarrow & ||\mathfrak{z}, m\rangle_\lambda
\end{array}$$

The paper is organized as follows. By reviewing some aspects of the two-particle Calogero-Sutherland model in section 2, we introduce new kind of PABGCSs $||\mathfrak{z}, m\rangle_\lambda$ and show their over completeness and resolution to the identity properties, will brought in section 4 deal with the general study of non classicality of these states. Section 3 includes detailed studies on NBGCSs and their properties. There to realize the resolution of the identity condition, we have found the positive definite measures on the complex plane and also their tendency to the well-known Barut-Girardello coherent states(BGCSs) [33] is reviewed. Some interesting features are found. For instance, we have shown that they can be considered as an eigenstate of certain annihilation operators and can be interpreted as NLCSs with special nonlinearity functions. Furthermore, it has been discussed in detail that they evolve in time as like as the canonical coherent states. In other words NBGCS possess the temporal stability property, too. We conclude in section 5.

2 Review and Construction

In our work we consider dynamics of a single bosonic mode, described by the Calogero-Sutherland Hamiltonian H^λ on the half-line x

$$H^\lambda = \frac{1}{2} \left[-\frac{d^2}{dx^2} + x^2 + \frac{\lambda(\lambda-1)}{x^2} \right]. \quad (1)$$

Here, the simple an-harmonic term $\frac{\lambda(\lambda-1)}{x^2}$ refers to the Goldman-Krivchenkov potential [38]. In Refs. [33, 39], it has been shown that the second-order differential operators

$$J_\pm^\lambda := \frac{1}{4} \left[\left(x \mp \frac{d}{dx} \right)^2 - \frac{\lambda(\lambda-1)}{x^2} \right], \quad (2)$$

$$J_3^\lambda := \frac{H^\lambda}{2}, \quad (3)$$

satisfy the standard commutation relations of $su(1,1)$ Lie algebra as follows

$$[J_+^\lambda, J_-^\lambda] = -2J_3^\lambda, \quad [J_3^\lambda, J_\pm^\lambda] = \pm J_\pm^\lambda. \quad (4)$$

Also, product the unitary and positive-integer IREP of $su(1,1)$ Lie algebra as

$$J_+^\lambda |n-1, \lambda\rangle = \sqrt{n \left(n + \lambda - \frac{1}{2} \right)} |n, \lambda\rangle, \quad (5)$$

$$J_-^\lambda |n, \lambda\rangle = \sqrt{n \left(n + \lambda - \frac{1}{2} \right)} |n-1, \lambda\rangle, \quad (6)$$

$$J_3^\lambda |n, \lambda\rangle = \left(n + \frac{\lambda}{2} + \frac{1}{4} \right) |n, \lambda\rangle. \quad (7)$$

We assume that the set of states described above form complete and orthonormal basis of an infinite dimensional Hilbert space, i.e.

$$\begin{aligned} \mathcal{H}^\lambda &:= \text{span}\{|n, \lambda\rangle | \langle n, \lambda | m, \lambda \rangle = \delta_{nm}\}_{n=0}^\infty, \\ \langle x | n, \lambda \rangle &:= (-1)^n \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\lambda+\frac{1}{2})}} x^\lambda e^{-\frac{x^2}{2}} L_n^{\lambda-\frac{1}{2}}(x^2), \quad \lambda > \frac{-1}{2}, \end{aligned} \quad (8)$$

where $L_n^{\lambda-\frac{1}{2}}(x)$ denotes the associated Laguerre functions [40]. Along with the orthogonality of the associated Laguerre polynomials, the orthogonality relation of the basis of \mathcal{H}^λ reads

$$\langle n, \lambda | m, \lambda \rangle := \frac{2n!}{\Gamma(n+\lambda+\frac{1}{2})} \int_0^\infty x^{2\lambda} e^{-x^2} L_n^{\lambda-\frac{1}{2}}(x^2) L_m^{\lambda-\frac{1}{2}}(x^2) dx = \delta_{nm}. \quad (9)$$

It is useful to stress that the two operators J_+^λ and J_-^λ are Hermitian conjugate of each others with respect to the inner product (9) and J_3^λ is self-adjoint operator, too. We commence

by establishing our formalism by collecting some well known facts about BGCSs, which are defined as the action of the B-G displacement operator $\mathcal{D}_{BG}(\mathfrak{z})$ on a vacuum state

$$|\mathfrak{z}, 0\rangle_\lambda \equiv \mathcal{D}_{BG}(\mathfrak{z})|0, \lambda\rangle \quad (10)$$

$$\mathcal{D}_{BG}(\mathfrak{z}) := e^{\frac{\mathfrak{z}}{\hat{N}+\lambda-\frac{1}{2}}J_+^\lambda} \quad (11)$$

$$\hat{N} := J_3^\lambda - \frac{\lambda}{2} - \frac{1}{4}, \quad \hat{N}|n, \lambda\rangle = n|n, \lambda\rangle \quad (12)$$

and, introduce the following modifications for a deformed setting

$$||\mathfrak{z}, m\rangle_\lambda := M_{m,\lambda}^{-\frac{1}{2}}(|\mathfrak{z}|)\mathcal{D}_{BG}(\mathfrak{z})|m, \lambda\rangle, \quad (13)$$

where $\mathfrak{z} = |\mathfrak{z}|e^{i\varphi}$, $0 \leq |\mathfrak{z}|$, $0 \leq \varphi \leq 2\pi$ is the coherence parameter. Clearly, the normalized states $||\mathfrak{z}, m\rangle_\lambda$ become standard Barut-Girardello coherent states for the Calogero-Sutherland model, $|z\rangle_{BG}^\lambda (= |\mathfrak{z}, 0\rangle_\lambda)$ (Eq. (6) in Ref. [33]), while m tends to zero. Putting

$$|m, \lambda\rangle = \frac{(J_+^\lambda)^m}{\sqrt{m!(\lambda + \frac{1}{2})_m}}|0, \lambda\rangle \quad (14)$$

into (13), we obtain

$$||\mathfrak{z}, m\rangle_\lambda = \frac{e^{\frac{\mathfrak{z}}{\hat{N}+\lambda-\frac{1}{2}}J_+^\lambda} (J_+^\lambda)^m}{\sqrt{m!(\lambda + \frac{1}{2})_m} M_{m,\lambda}(|\mathfrak{z}|)}|0, \lambda\rangle, \quad (15)$$

where $(a)_m = \frac{\Gamma(a+m)}{\Gamma(a)}$ denotes the Pochammers notation. The above expression after restoring the formula

$$\begin{aligned} \frac{\mathfrak{z}}{\hat{N} + \lambda - \frac{1}{2}} J_+^\lambda &= J_+^\lambda \frac{\mathfrak{z}}{\hat{N} + \lambda - \frac{1}{2} + 1}, \\ \frac{\mathfrak{z}}{\hat{N} + \lambda - \frac{1}{2}} (J_+^\lambda)^2 &= (J_+^\lambda)^2 \frac{\mathfrak{z}}{\hat{N} + \lambda - \frac{1}{2} + 2}, \\ &\vdots \\ &\Downarrow \\ e^{\frac{\mathfrak{z}}{\hat{N}+\lambda-\frac{1}{2}}J_+^\lambda} (J_+^\lambda)^m &= (J_+^\lambda)^m e^{\frac{\mathfrak{z}}{\hat{N}+\lambda-\frac{1}{2}+m}J_+^\lambda}, \end{aligned} \quad (16)$$

yields

$$\begin{aligned} ||\mathfrak{z}, m\rangle_\lambda &= \frac{(J_+^\lambda)^m}{\sqrt{m!(\lambda + \frac{1}{2})_m} M_{m,\lambda}(|\mathfrak{z}|)} e^{\frac{\mathfrak{z}}{\hat{N}+\lambda-\frac{1}{2}+m}J_+^\lambda} |0, \lambda\rangle \\ &= \frac{(J_+^\lambda)^m}{\sqrt{m! (\lambda + \frac{1}{2})_m \frac{M_{m,\lambda}(|\mathfrak{z}|)}{\mathfrak{M}_{m,\lambda}(|\mathfrak{z}|)}}} |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda, \end{aligned} \quad (17)$$

here \mathfrak{f}_m , that will be determined later, is an operator-valued function of the number operator \hat{N} and $\mathfrak{M}_{m,\lambda}(|\mathfrak{z}|)$ is chosen so that $|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$ is normalized, i.e. ${}_\lambda\langle\mathfrak{z}, \mathfrak{f}_m|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda = 1$. At this stage, we will postpone investigation of the properties of $||\mathfrak{z}, m\rangle_\lambda$ up to next section and consciously concentrate to solve a problem that how the states $|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$ can be recognized as nonlinear type of coherent states.

3 NBGCSs And It's Properties

In this section we want to inform that the states $|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$ were introduced above as *NBGCs*, can be categorized as special class of nonlinear coherent states. For this reason we will set up detailed studies on statistical properties of them. Proportional nonlinear function associated to *NBGCs* are introduced, also to analyze their statistical behavior some of the characters including the second-order correlation function, Mandel's parameter and squeezing factor are computed.

◇ NBGCS

Let $|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$ denotes the states generated by m -deformed Barut-Girardello displacement operator $\mathcal{D}_{BG}^m(\mathfrak{z})$, i.e.

$$|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda := \mathfrak{M}_{m,\lambda}^{-\frac{1}{2}}(|\mathfrak{z}|) \mathcal{D}_{BG}^m(\mathfrak{z})|0, \lambda\rangle, \quad m \in N_0, \quad (18)$$

$$\mathcal{D}_{BG}^m(\mathfrak{z}) := e^{\frac{\mathfrak{z}}{\hat{N} + \lambda - \frac{1}{2} + m} J_+^\lambda}, \quad \lim_{m \rightarrow 0} \mathcal{D}_{BG}^m(\mathfrak{z}) = \mathcal{D}_{BG}(\mathfrak{z}). \quad (19)$$

Inserting (14) into (18) and using the identity

$$\left(\frac{1}{\hat{N} + \lambda - \frac{1}{2} + m} J_+^\lambda \right)^n = (J_+^\lambda)^n \frac{1}{\left(\hat{N} + \lambda + \frac{1}{2} + m \right)_n}, \quad (20)$$

we obtain

$$|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda = \mathfrak{M}_{m,\lambda}^{-\frac{1}{2}}(|\mathfrak{z}|) \sum_{n=0}^{\infty} \frac{\mathfrak{z}^n}{\left(\lambda + \frac{1}{2} + m \right)_n} \sqrt{\frac{\left(\lambda + \frac{1}{2} \right)_n}{n!}} |n, \lambda\rangle. \quad (21)$$

Due to the orthogonality relation (9) it follows that overlapping of two different kinds of these normalized states must be nonorthogonal, if $m' \neq m$ and $\mathfrak{z}' \neq \mathfrak{z}$, i.e.

$${}_\lambda \langle \mathfrak{z}', \mathfrak{f}_{m'} | \mathfrak{z}, \mathfrak{f}_m \rangle_\lambda = \frac{{}_1F_2 \left(\left[\lambda + \frac{1}{2} \right], \left[\lambda + \frac{1}{2} + m', \lambda + \frac{1}{2} + m \right], \overline{\mathfrak{z}} \mathfrak{z} \right)}{\sqrt{\mathfrak{M}_{m'}^\lambda(|\mathfrak{z}'|) \mathfrak{M}_m^\lambda(|\mathfrak{z}|)}}. \quad (22)$$

Then, $\mathfrak{M}_{m,\lambda}(|\mathfrak{z}|)$ can be calculated to be taken as

$$\mathfrak{M}_{m,\lambda}(|\mathfrak{z}|) = {}_1F_2 \left(\left[\lambda + \frac{1}{2} \right], \left[\lambda + \frac{1}{2} + m, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right). \quad (23)$$

From the completeness relation of the Fock space states, straightforwardly, resolution of the identity

$$\oint_{\mathbb{C}(\mathfrak{z})} |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda {}_\lambda \langle \mathfrak{z}, \mathfrak{f}_m| d\mu_{m,\lambda}(|\mathfrak{z}|) = I^\lambda = \sum_{n=0}^{\infty} |n, \lambda\rangle \langle n, \lambda|, \quad (24)$$

is realized for the states $|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$ with respect to the appropriate measure, $d\mu_{m,\lambda}(|\mathfrak{z}|) := \mathfrak{K}_{m,\lambda}(|\mathfrak{z}|) \frac{d|\mathfrak{z}|^2}{2} d\varphi$, that relates it to the Meijer's G-function(see $\frac{7-811}{4}$ in [40]):

$$\mathfrak{K}_{m,\lambda}(|\mathfrak{z}|) = \frac{\mathfrak{M}_{m,\lambda}(|\mathfrak{z}|)}{\pi \left(\lambda + \frac{1}{2} \right)_m \Gamma\left(\lambda + \frac{1}{2} + m \right)} \mathbf{G}_{24}^{31} \left(|\mathfrak{z}|^2 \middle| \begin{smallmatrix} 0, \lambda - \frac{1}{2} \\ 0, \lambda - \frac{1}{2} + m, \lambda - \frac{1}{2} + m, 0 \end{smallmatrix} \right), \quad \lambda - \frac{1}{2} \in 2\mathbb{N}_0 \quad (25)$$

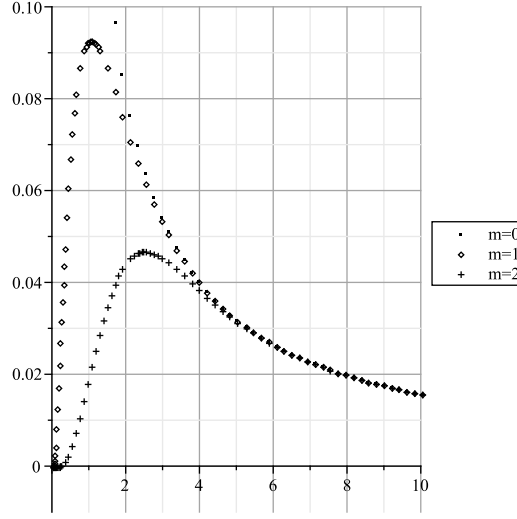


Figure 1: Plots of the non-oscillating measures $\mathfrak{K}_{m,\lambda}(|\mathfrak{z}|)$ in terms of $|\mathfrak{z}|$ for different values of m and $\lambda = \frac{1}{2}$. The dotted curve corresponds to the standard B-G coherent states.

It is worth to mention that $\mathfrak{K}_{m=0,\lambda}(|\mathfrak{z}|)$ reduces to the well known positive definite measure of the standard Barut-Girardello coherent states, i.e.

$$\lim_{m \rightarrow 0} \mathfrak{K}_{m,\lambda}(|\mathfrak{z}|) = \frac{2}{\pi} I_{\lambda-\frac{1}{2}}(2|\mathfrak{z}|) K_{\lambda-\frac{1}{2}}(2|\mathfrak{z}|),$$

where $I_a(x)$ and $K_b(x)$ refer to the modified Bessel functions of the first and second kinds, respectively(see Figure 1).

◇ *Time Evolution*

Based on the relations (3) and (8), we have

$$H^\lambda |n, \lambda\rangle = (2n + \lambda + \frac{1}{2}) |n, \lambda\rangle. \quad (26)$$

Then the states (21) evolve in time as

$$\begin{aligned} e^{-itH^\lambda} |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda &= e^{-it(\lambda+\frac{1}{2})} \mathfrak{M}_{m,\lambda}^{-\frac{1}{2}}(|\mathfrak{z}|) \sum_{n=0}^{\infty} \frac{(\mathfrak{z}e^{-i2t})^n}{(\lambda + \frac{1}{2} + m)_n} \sqrt{\frac{(\lambda + \frac{1}{2})_n}{n!}} |n, \lambda\rangle \\ &= e^{-it(\lambda+\frac{1}{2})} |\mathfrak{z}e^{-i2t}, \mathfrak{f}_m\rangle_\lambda. \end{aligned} \quad (27)$$

In fact, these states are temporally stable.

◇ *Coordinate Representation of $|\mathfrak{z}, \mathfrak{f}_n\rangle_\lambda$*

Based on a new expression of the Laguerre polynomials as an operator-valued function given in [41]

$$L_n^\alpha(y) = \frac{1}{n!} y^{-\alpha} \left(\frac{d}{dy} - 1 \right)^n y^{n+\alpha}, \quad (28)$$

also, according to Eqs. (8) and (21) we have

$$\begin{aligned}\langle x|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda &= \sqrt{\frac{2}{\Gamma(\lambda + \frac{1}{2})\mathfrak{M}_{m,\lambda}}}(|\mathfrak{z}|) \sum_{n=0}^{\infty} \frac{(-\mathfrak{z})^n}{(\lambda + \frac{1}{2} + m)_n} x^\lambda e^{-\frac{x^2}{2}} L_n^{\lambda-\frac{1}{2}}(x^2) \\ &= \sqrt{\frac{2}{\Gamma(\lambda + \frac{1}{2})\mathfrak{M}_{m,\lambda}}}(|\mathfrak{z}|) \sum_{n=0}^{\infty} \frac{(-\mathfrak{z})^n}{n! (\lambda + \frac{1}{2} + m)_n} e^{-\frac{y}{2}} y^{\frac{1-\lambda}{2}} \left(\frac{d}{dy} - 1\right)^n y^{n+\lambda-\frac{1}{2}}|_{y=x^2}.\end{aligned}$$

If we resort

$$\left(\frac{d}{dy} - 1\right)^n y^n = \left(y\frac{d}{dy} + n - y\right) \dots \left(y\frac{d}{dy} + 1 - y\right) = \left(y\frac{d}{dy} - y + 1\right)_n,$$

it becomes

$$\begin{aligned}\langle x|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda &= \sqrt{\frac{2e^{-y}y^{1-\lambda}}{\Gamma(\lambda + \frac{1}{2})\mathfrak{M}_{m,\lambda}(|\mathfrak{z}|)}} \sum_{n=0}^{\infty} \frac{\left(y\frac{d}{dy} - y + 1\right)_n (-\mathfrak{z})^n}{n! (\lambda + \frac{1}{2} + m)_n} y^{\lambda-\frac{1}{2}}|_{y=x^2} \\ &= \sqrt{\frac{2e^{-y}y^{1-\lambda}}{\Gamma(\lambda + \frac{1}{2})}} \frac{{}_1F_1\left(\left[y\frac{d}{dy} - y + 1\right], \left[\lambda + \frac{1}{2} + m\right], -\mathfrak{z}\right) y^{\lambda-\frac{1}{2}}}{\sqrt{{}_1F_2\left(\left[\lambda + \frac{1}{2}\right], \left[\lambda + \frac{1}{2} + m, \lambda + \frac{1}{2} + m\right], |\mathfrak{z}|^2\right)}}|_{y=x^2},\end{aligned}\quad (29)$$

where ${}_1F_1$ is the (Kummer's) confluent hypergeometric function which corresponds to the special case $u = v = 1$ of the generalized hypergeometric function ${}_uF_v$ (with u numerator and v denominator parameters). For instance, the explicit compact forms of $|\mathfrak{z}, \mathfrak{f}_0\rangle_\lambda$ reduces to the standard Barut-Girardello coherent states [33]

$$\langle x|\mathfrak{z}, \mathfrak{f}_0\rangle_\lambda = \sqrt{2x} \left(-\frac{\mathfrak{z}}{|\mathfrak{z}|}\right)^{\frac{1}{4}-\frac{\lambda}{2}} \frac{J_{\lambda-\frac{1}{2}}(2ix\sqrt{\mathfrak{z}})}{\sqrt{I_{\lambda-\frac{1}{2}}(2|\mathfrak{z}|)}} e^{-\mathfrak{z}-\frac{x^2}{2}} \quad (30)$$

◇ **Nonlinearity function**

Using the relations (6) and (21), one can show that

$$\begin{aligned}J_-^\lambda |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda &= \mathfrak{M}_{m,\lambda}^{-\frac{1}{2}}(|\mathfrak{z}|) \sum_{n=0}^{\infty} \left[\mathfrak{z} \frac{\lambda + \frac{1}{2} + n}{\lambda + \frac{1}{2} + m + n} \right] \frac{\mathfrak{z}^n}{(\lambda + \frac{1}{2} + m)_n} \sqrt{\frac{(\lambda + \frac{1}{2})_n}{n!}} |n, \lambda\rangle \\ &\quad \Downarrow \\ \left(1 + \frac{m}{\hat{N} + \lambda + \frac{1}{2}}\right) J_-^\lambda |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda &= \mathfrak{z} |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda.\end{aligned}\quad (31)$$

This formula specifies NBGCSs as new class of nonlinear coherent states with characterized nonlinearity functions, $\mathfrak{f}_m \left(= 1 + \frac{m}{\hat{N} + \lambda + \frac{1}{2}}\right)$, which tend to unity for $m = 0$.

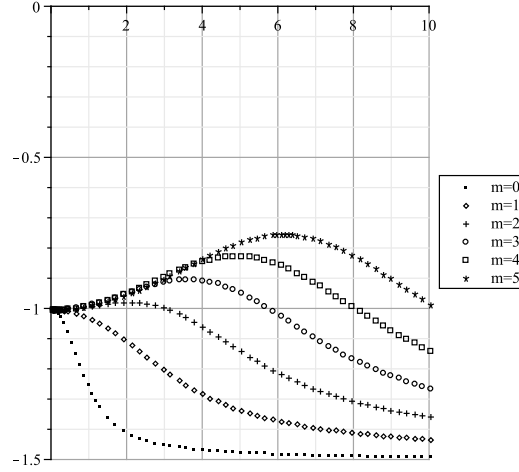


Figure 2: Graphs of the Mandel's parameters $\mathfrak{Q}_m^\lambda(|\mathfrak{z}|)$ in the NBGCSs, versus $|\mathfrak{z}|$ for different values of m and $\lambda = \frac{1}{2}$. The dotted curve corresponds to the standard B-G coherent states.

◇ *Anti-bunching effect and sub-Poissonian statistics*

Now we are in a position to study the anti-bunching effect as well as the statistics of NBGCSs. We introduce the second-order correlation function for these states

$$(g^{(2)})_m^\lambda(|\mathfrak{z}|) = \frac{\langle \hat{N}^2 \rangle_m^\lambda - \langle \hat{N} \rangle_m^\lambda^2}{\langle \hat{N} \rangle_m^\lambda^2}. \quad (32)$$

Furthermore, the inherent statistical properties of the GNCSs follows also from calculating the Mandel parameter ¹

$$\mathfrak{Q}_m^\lambda(|\mathfrak{z}|) = \langle \hat{N} \rangle_m^\lambda \left[(g^{(2)})_m^\lambda(|\mathfrak{z}|) - 1 \right]. \quad (33)$$

In order to find the function $(g^{(2)})_m^\lambda(|\mathfrak{z}|)$, also Mandel's parameter $\mathfrak{Q}_m^\lambda(|\mathfrak{z}|)$, let us begin with the expectation values of the number operator \hat{N} and it's square in the basis of the Fock states $|n, \lambda\rangle$

$$\begin{aligned} \langle \hat{N} \rangle_m^\lambda &= |\mathfrak{z}|^2 \left[\frac{\lambda + \frac{1}{2}}{(\lambda + \frac{1}{2} + m)^2} \right] \frac{{}_1F_2 \left(\left[\lambda + \frac{3}{2} \right], \left[\lambda + \frac{3}{2} + m, \lambda + \frac{3}{2} + m \right], |\mathfrak{z}|^2 \right)}{{}_1F_2 \left(\left[\lambda + \frac{1}{2} \right], \left[\lambda + \frac{1}{2} + m, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)}, \\ \langle \hat{N}^2 \rangle_m^\lambda &= |\mathfrak{z}|^4 \left[\frac{(\lambda + \frac{1}{2})(\lambda + \frac{3}{2})}{(\lambda + \frac{1}{2} + m)^2(\lambda + \frac{3}{2} + m)^2} \right] \\ &\quad \times \frac{{}_1F_2 \left(\left[\lambda + \frac{5}{2} \right], \left[\lambda + \frac{5}{2} + m, \lambda + \frac{5}{2} + m \right], |\mathfrak{z}|^2 \right)}{{}_1F_2 \left(\left[\lambda + \frac{1}{2} \right], \left[\lambda + \frac{1}{2} + m, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)}. \end{aligned}$$

¹A state for which $\mathfrak{Q}_m^\lambda(|\mathfrak{z}|) > 0$ (or $(g^{(2)})_m^\lambda(|\mathfrak{z}|) > 1$) is called super-Poissonian (bunching effect), if $\mathfrak{Q} = 0$ (or $g^{(2)} = 1$) the state is called Poissonian, while a state for which $\mathfrak{Q} < 0$ (or $g^{(2)} < 1$) is, also, called sub-Poissonian (antibunching effect).

Our calculations show that for any case $m \in \mathbb{N}_0$ as well as $\lambda \in 2\mathbb{N}_0 + \frac{1}{2}$, the Mandel's parameters are really smaller than zero. In other words the *NBGCs* exhibit fully anti-bunching effects, or sub-Poissonian statistics. As shown in figure 2, $\mathfrak{Q}_m^\lambda(|\mathfrak{z}|)$ have been plotted in terms of $|\mathfrak{z}|$ for several values of $m (= 0, 1, 2, 3, 4 \text{ and } 5)$ where we choose $\lambda = \frac{1}{2}$.

◇ *SU(1, 1) squeezing*

We introduce two generalized Hermitian quadrature operators $X_{1(2)}^\lambda$

$$X_1^\lambda = \frac{J_+^\lambda + J_-^\lambda}{2}, \quad X_2^\lambda = \frac{J_-^\lambda - J_+^\lambda}{2i}, \quad (34)$$

with the commutation relation $[X_1^\lambda, X_2^\lambda] = iJ_3^\lambda$. From this, the uncertainty condition for the variances of the quadratures X_i follow

$$\langle (\Delta X_1^\lambda)^2 \rangle \langle (\Delta X_2^\lambda)^2 \rangle \geq \frac{|\langle J_3^\lambda \rangle|^2}{4}, \quad (35)$$

where $\langle (\Delta X_i^\lambda)^2 \rangle (= \langle X_i^{\lambda^2} \rangle - \langle X_i^\lambda \rangle^2)$ have been expressed as

$$\langle (\Delta X_{1(2)}^\lambda)^2 \rangle = \frac{2 \langle J_+^\lambda J_-^\lambda \rangle + 2 \langle J_3^\lambda \rangle \pm \langle J_+^{\lambda^2} + J_-^{\lambda^2} \rangle - \langle J_-^\lambda \pm J_+^\lambda \rangle^2}{4}. \quad (36)$$

Here, the angular brackets denote averaging over an arbitrary normalizable state for which the mean values are well defined, i.e.

$$\langle X_i \rangle = {}_\lambda \langle \mathfrak{z}, \mathfrak{f}_m | X_i | \mathfrak{z}, \mathfrak{f}_m \rangle_\lambda.$$

For instance, we have the relations

$$\begin{aligned} \langle J_+^\lambda \rangle &= \overline{\langle J_-^\lambda \rangle} = \mathfrak{z} \left[\frac{\lambda + \frac{1}{2}}{\lambda + \frac{1}{2} + m} \right] \frac{{}_1F_2 \left(\left[\lambda + \frac{3}{2} \right], \left[\lambda + \frac{3}{2} + m, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)}{{}_1F_2 \left(\left[\lambda + \frac{1}{2} \right], \left[\lambda + \frac{1}{2} + m, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)}, \\ \langle J_+^{\lambda^2} \rangle &= \overline{\langle J_-^{\lambda^2} \rangle} = \mathfrak{z}^2 \left[\frac{(\lambda + \frac{1}{2})(\lambda + \frac{3}{2})}{(\lambda + \frac{1}{2} + m)(\lambda + \frac{3}{2} + m)} \right] \\ &\quad \times \frac{{}_1F_2 \left(\left[\lambda + \frac{5}{2} \right], \left[\lambda + \frac{5}{2} + m, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)}{{}_1F_2 \left(\left[\lambda + \frac{1}{2} \right], \left[\lambda + \frac{1}{2} + m, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)}, \\ \langle J_+^\lambda J_-^\lambda \rangle &= |\mathfrak{z}|^2 \left[\frac{(\lambda + \frac{1}{2})\Gamma(\lambda + \frac{1}{2} + m)}{\Gamma(\lambda + \frac{3}{2} + m)} \right]^2 \\ &\quad \times \frac{{}_2F_3 \left(\left[\lambda + \frac{3}{2}, \lambda + \frac{3}{2} \right], \left[\lambda + \frac{1}{2}, \lambda + \frac{3}{2} + m, \lambda + \frac{3}{2} + m \right], |\mathfrak{z}|^2 \right)}{{}_1F_2 \left(\left[\lambda + \frac{1}{2} \right], \left[\lambda + \frac{1}{2} + m, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)}, \\ \langle J_3^\lambda \rangle &= |\mathfrak{z}|^2 \left[\frac{\lambda + \frac{1}{2}}{(\lambda + \frac{1}{2} + m)^2} \right] \frac{{}_1F_2 \left(\left[\lambda + \frac{3}{2} \right], \left[\lambda + \frac{3}{2} + m, \lambda + \frac{3}{2} + m \right], |\mathfrak{z}|^2 \right)}{{}_1F_2 \left(\left[\lambda + \frac{1}{2} \right], \left[\lambda + \frac{1}{2} + m, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)} \\ &\quad + \frac{\lambda}{2} + \frac{1}{4}. \end{aligned}$$

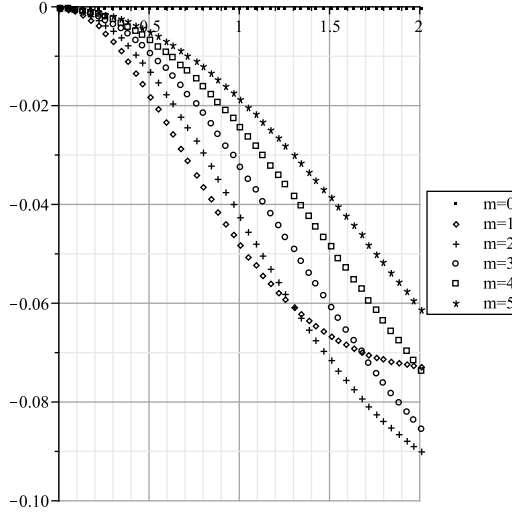


Figure 3: Squeezing factors in the field quadratures X_1^λ in the NBGCSs, in terms of $|\mathfrak{z}|$ for different values of m and $\lambda = \frac{1}{2}$ while we choose the phase $\varphi = \frac{\pi}{3}$. The dotted curve corresponds the standard B-G coherent states ($\mathcal{S}_1^{m=0,\lambda} = 0$).

They result that, $\langle(\Delta X_{1(2)}^\lambda)^2\rangle$ for any value of λ , are efficiently dependent on the complex variable \mathfrak{z} and the deformation parameter m .

Following Walls (1983) we will say that the state is $SU(1,1)$ squeezed if the following condition is fulfilled [42]

$$\langle(\Delta X_i^\lambda)^2\rangle < \frac{|\langle J_3^\lambda \rangle|}{2}, \quad \text{for } i = 1 \text{ or } 2, \quad (37)$$

or, with respect to the squeezing factor $\mathcal{S}_i^{m,\lambda}(\mathfrak{z})$ as [43]

$$\mathcal{S}_i^{m,\lambda}(\mathfrak{z}) \left(= 2 \frac{\langle(\Delta X_i^\lambda)^2\rangle}{|\langle J_3^\lambda \rangle|} - 1 \right) < 0, \quad (38)$$

however maximally squeezing is obtained for $\mathcal{S}_i^{m,\lambda}(\mathfrak{z}) = -1$. We illustrate in figure 3 squeezing factors $\mathcal{S}_1^{m,\lambda}(\mathfrak{z})$ as functions of $|\mathfrak{z}|$ for different values of m ($= 0, 1, 2, 3, 4$ and 5) where we choose the phase $\varphi = \frac{\pi}{3}$ as well as $\lambda = \frac{1}{2}$. They become really smaller than zero for any values of $|\mathfrak{z}|$ and we find that by increasing m the degree of squeezing is enhanced. However we will loose the squeezing in X_1 when m reaches zero. In other words, for the case $m = 0$ (standard BG coherent states) we would not expect to take squeezing neither in X_1 nor in X_2 quadratures.

4 PABGCSs

According to (17), we introduce the state $||\mathfrak{z}, \mathbf{m}\rangle_\lambda$ defined by

$$||\mathfrak{z}, m\rangle_\lambda = \frac{(J_+^\lambda)^m}{\sqrt{m! \left(\lambda + \frac{1}{2}\right)_m \frac{M_{m,\lambda}(|\mathfrak{z}|)}{\mathfrak{M}_{m,\lambda}(|\mathfrak{z}|)}}} |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda, \quad (39)$$

where $|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$ refer to the NBGCSs. In the limit $\mathfrak{z} \rightarrow 0$ and $m \rightarrow 0$ the states $||\mathfrak{z}, m\rangle_\lambda$ reduce the Fock state and the standard Barut-Girardello coherent state, respectively. Thus, it is a state intermediate Fock state and BGCS, so we may call such states as **Photon Added Barut-Girardello Coherent States**.

Substituting of (21) into the right hand side of (17), leads to the state $||\mathfrak{z}, \mathbf{m}\rangle_\lambda$ in terms of the Fock states as

$$\begin{aligned} ||\mathfrak{z}, \mathbf{m}\rangle_\lambda &= \frac{(J_+^\lambda)^m}{\sqrt{m! \left(\lambda + \frac{1}{2}\right)_m M_{m,\lambda}(|\mathfrak{z}|)}} \sum_{n=0}^{\infty} \frac{\mathfrak{z}^n}{\left(\lambda + \frac{1}{2} + m\right)_n} \sqrt{\frac{\left(\lambda + \frac{1}{2}\right)_n}{n!}} |n, \lambda\rangle \\ &= \frac{\sum_{n=0}^{\infty} \frac{\mathfrak{z}^n}{\left(\lambda + \frac{1}{2} + m\right)_n} \sqrt{\frac{\left(\lambda + \frac{1}{2}\right)_n (n+m)! (n+\lambda - \frac{1}{2} + m)!}{n!^2 (n+\lambda - \frac{1}{2})!}}}{\sqrt{m! \left(\lambda + \frac{1}{2}\right)_m M_{m,\lambda}(|\mathfrak{z}|)}} |n+m, \lambda\rangle \\ &= \frac{1}{\sqrt{M_{m,\lambda}(|\mathfrak{z}|)}} \sum_{n=0}^{\infty} \frac{\mathfrak{z}^n}{n!} \sqrt{\frac{(m+1)_n}{\left(\lambda + \frac{1}{2} + m\right)_n}} |n+m, \lambda\rangle. \end{aligned} \quad (40)$$

At the same time

$$M_{m,\lambda}(|\mathfrak{z}|) = {}_1F_2 \left([m+1], \left[1, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right), \quad (41)$$

where the latter results from the requirement ${}_ \lambda \langle \mathfrak{z}, \mathbf{m} | \mathfrak{z}, \mathbf{m} \rangle_\lambda = 1$ and lead to the following non-vanishing scalar products

$$\begin{aligned} {}_\lambda \langle \mathfrak{z}', \mathbf{m}' | \mathfrak{z}, \mathbf{m} \rangle_\lambda &= \frac{\left[\frac{(1)_m \left(\lambda + \frac{1}{2}\right)_{m'}}{(1)_{m'} \left(\lambda + \frac{1}{2}\right)_m} \right]^{\frac{1}{2}} \mathfrak{z}^{m-m'} \sum_{n=0}^{\infty} \frac{|\mathfrak{z}|^{2n}}{n!} \frac{(m+1)_n}{\left(\lambda + \frac{1}{2} + m\right)_n (n+m-m')!}}{\sqrt{{}_1F_2 \left([m'+1], \left[1, \lambda + \frac{1}{2} + m' \right], |\mathfrak{z}'|^2 \right) {}_1F_2 \left([m+1], \left[1, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)}}, \\ {}_\lambda \langle \mathfrak{z}', \mathbf{m} | \mathfrak{z}, \mathbf{m} \rangle_\lambda &= \frac{{}_1F_2 \left([m+1], \left[1, \lambda + \frac{1}{2} + m \right], \overline{\mathfrak{z}} \mathfrak{z} \right)}{\sqrt{{}_1F_2 \left([m+1], \left[1, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}'|^2 \right) {}_1F_2 \left([m+1], \left[1, \lambda + \frac{1}{2} + m \right], |\mathfrak{z}|^2 \right)}}. \end{aligned}$$

◇ Resolution of unity (or completeness)

From equation (40) we see that the state $||\mathfrak{z}, \mathbf{m}\rangle_\lambda$ is a linear combination of all number states

starting with $n = m$. In other words, the first m number states $n = 0, 1, \dots, m-1$, are absent from these state. Then, the unity operator in this space is to be written as

$$I_m^\lambda = \sum_{n=m}^{\infty} |n, \lambda\rangle \langle n, \lambda| = \sum_{n=0}^{\infty} |n+m, \lambda\rangle \langle n+m, \lambda|. \quad (42)$$

Evidently, in the right-hand side of the above equation, the identity operator on the full Hilbert space does not appear, because of the initial m states of the basis set vectors are omitted. This leads to the following resolution of unity via bounded, positive definite and non-oscillating measures, $d\eta_m(|\mathfrak{z}|) := K_m(|\mathfrak{z}|) \frac{d|\mathfrak{z}|^2}{2} d\varphi$,

$$\oint_{\mathbb{C}(\mathfrak{z})} ||\mathfrak{z}, \mathbf{m}\rangle_\lambda {}_\lambda \langle \mathfrak{z}, \mathbf{m}| d\eta_{m,\lambda}(|\mathfrak{z}|) = I_m^\lambda, \quad (43)$$

$$K_{m,\lambda}(|\mathfrak{z}|) = \frac{\Gamma(m+1)}{2\pi\Gamma(\lambda + \frac{1}{2} + m)} M_{m,\lambda}(|\mathfrak{z}|) \mathbf{G}_{24}^{31} \left(|\mathfrak{z}|^2 \mid \begin{matrix} 0, m \\ 0, 0, \lambda - \frac{1}{2} + m, 0 \end{matrix} \right). \quad (44)$$

◇ *Construction of Nonlinearity function*

In this section we construct the explicit form of the operator valued of nonlinearity function associated to the PABGCS. Since, the coherent states $|\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$ satisfy following eigenvalue equation

$$\left(1 + \frac{m}{\hat{N} + \lambda + \frac{1}{2}}\right) J_-^\lambda |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda = \mathfrak{z} |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda,$$

so, multiplying both sides of this equation by $(J_+^\lambda)^m$ yields

$$(J_+^\lambda)^m \left(1 + \frac{m}{\hat{N} + \lambda + \frac{1}{2}}\right) J_-^\lambda |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda = (J_+^\lambda)^m \mathfrak{z} |\mathfrak{z}, \mathfrak{f}_m\rangle_\lambda$$

which, making use of the commutation relations (4) and the identities

$$\begin{aligned} (J_+^\lambda)^m J_-^\lambda &= J_-^\lambda (J_+^\lambda)^m - 2m \left(\hat{N} + \frac{\lambda}{2} + \frac{3}{4} - \frac{m}{2} \right) (J_+^\lambda)^{m-1}, \\ \frac{1}{(\hat{N} + 1)(\hat{N} + \lambda + \frac{1}{2})} J_-^\lambda J_+^\lambda &= 1, \end{aligned}$$

leads to

$$\frac{\hat{N} + \lambda + \frac{1}{2}}{\hat{N} + \lambda + \frac{1}{2} - m} \left(1 - \frac{2m \left(\hat{N} + \frac{\lambda}{2} + \frac{3}{4} - \frac{m}{2} \right)}{(\hat{N} + 1)(\hat{N} + \lambda + \frac{1}{2})}\right) J_-^\lambda ||\mathfrak{z}, \mathbf{m}\rangle_\lambda = \mathfrak{z} ||\mathfrak{z}, \mathbf{m}\rangle_\lambda, \quad (45)$$

and pretends them as nonlinear coherent states by the expression for nonlinearity function as

$$\frac{\hat{N} + \lambda + \frac{1}{2}}{\hat{N} + \lambda + \frac{1}{2} - m} \left(1 - \frac{2m \left(\hat{N} + \frac{\lambda}{2} + \frac{3}{4} - \frac{m}{2} \right)}{(\hat{N} + 1)(\hat{N} + \lambda + \frac{1}{2})}\right). \quad (46)$$

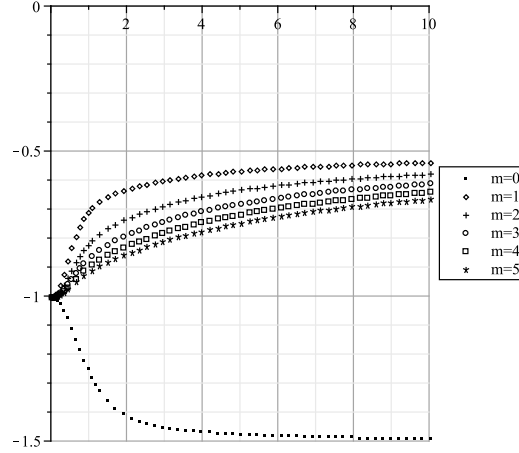


Figure 4: Mandel's parameters $Q_m^\lambda(|\mathfrak{z}|)$ in the PABGCSs, versus $|\mathfrak{z}|$ for different values of m and $\lambda = \frac{1}{2}$. The dotted curve corresponds to the standard B-G coherent states.

Obviously, it transforms to the identity operator for $m = 0$.

◇ *Sub-Poissonian Distribution For The Field In PABGCSs*

In this part we calculate quasi-probability distribution functions for the states PABGCS, which provide a convenient way of studying the nonclassical properties of the field. For this reason we begin to calculate the expectation values of the number operator \hat{N} and its square in the basis of the Fock states

$$\begin{aligned}\langle \hat{N} \rangle_m^\lambda &= m \frac{{}_2F_3([m+1, m+1], [1, m, \lambda + \frac{1}{2} + m], |\mathfrak{z}|^2)}{{}_1F_2([m+1], [1, \lambda + \frac{1}{2} + m], |\mathfrak{z}|^2)}, \\ \langle \hat{N}^2 \rangle_m^\lambda &= m^2 \frac{{}_3F_4([m+1, m+1, m+1], [1, m, m, \lambda + \frac{1}{2} + m], |\mathfrak{z}|^2)}{{}_1F_2([m+1], [1, \lambda + \frac{1}{2} + m], |\mathfrak{z}|^2)},\end{aligned}$$

which result the Mandel parameter $Q_m^\lambda(|\mathfrak{z}|)$ for these states as follow:

$$Q_m^\lambda(|\mathfrak{z}|) = \frac{\langle \hat{N}^2 \rangle_m^\lambda - \langle \hat{N} \rangle_m^\lambda{}^2}{\langle \hat{N} \rangle_m^\lambda} - 1. \quad (47)$$

Because of the structure of this function as illustrated in figure 4, the states $|\mathfrak{z}, \mathbf{m}\rangle_\lambda$ show sub-Poissonian statistics(or fully anti-bunching effects).

◇ *Squeezing Properties*

Our final step is to reveal that measurements on the states $|\mathfrak{z}, \mathbf{m}\rangle_\lambda$ come with squeezing for the field quadrature operators $X_{1(2)}^\lambda$. In this case, squeezing factors

$$S_{1(2)}^{m,\lambda} = \frac{2\langle (\Delta X_{1(2)}^\lambda)^2 \rangle}{|\langle J_3^\lambda \rangle|} - 1,$$

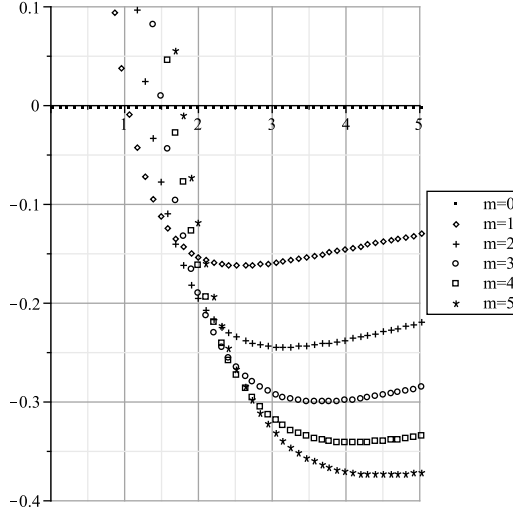


Figure 5: Squeezing in X_1^λ quadratures in the PABGCSs, against $|z|$ for $\varphi = 0, \lambda = \frac{1}{2}$ and different values of m . The dotted curve corresponds the standard B-G coherent states ($S_1^{m=0, \lambda} = 0$).

are coming with respect to the following expectation values

$$\begin{aligned} \langle J_+^\lambda \rangle &= \overline{\langle J_-^\lambda \rangle} = \bar{z}(m+1) \frac{{}_1F_2([m+2], [2, \lambda + \frac{1}{2} + m], |z|^2)}{{}_1F_2([m+1], [1, \lambda + \frac{1}{2} + m], |z|^2)}, \\ \langle J_+^{\lambda^2} \rangle &= \overline{\langle J_-^{\lambda^2} \rangle} = \bar{z}^2 \frac{(m+1)(m+2)}{2} \frac{{}_1F_2([m+3], [3, \lambda + \frac{1}{2} + m], |z|^2)}{{}_1F_2([m+1], [1, \lambda + \frac{1}{2} + m], |z|^2)}, \\ \langle J_+^\lambda J_-^\lambda \rangle &= m \left(m + \lambda - \frac{1}{2} \right) \frac{{}_2F_4([m+1, m+1], [1, m, \lambda - \frac{1}{2} + m], |z|^2)}{{}_1F_2([m+1], [1, \lambda + \frac{1}{2} + m], |z|^2)}, \\ \langle J_3^\lambda \rangle &= m \frac{{}_2F_3([m+1, m+1], [1, m, \lambda + \frac{1}{2} + m], |z|^2)}{{}_1F_2([m+1], [1, \lambda + \frac{1}{2} + m], |z|^2)} + \frac{\lambda}{2} + \frac{1}{4}. \end{aligned}$$

Figure 5 implies that the squeezing in X_1 arises when m is increasing, but it is disappeared while $m \rightarrow 0$.

5 Conclusions

Based on the process proposed in this presentation, it will be possible to reproduce broad range of states that are called nonlinear coherent states through of the generalized displacement operators. In a general view the formalism presented here provides a unified approach to construct all the employed CSs already introduced in different ways(the Barut-Girardello and nonlinear coherent states). These states realize a resolution of the identity with respect to positive definite measures on the complex plane. Finally, non-classicality properties of such states have been reviewed in detail. For instance, we have shown that their squeezing in X_1^λ quadratures are really considerable. In other words the system can be prepared in any excited Fock states so that $\mathcal{S}_{1(2)}^{m \geq 1, \lambda} < 0$.

As significant pictures of the above mentioned approach:

- *We model quantum mechanical system which follows fully sub-poissonian statistics.*
- *Performance of a strategy for finding considerable as well as controllable squeezing properties in field quadratures.*
- *Non-classical features of these states could be strictly controlled by energy number.*

We assert that our approach, has the potentiality to be used for the construction of a variety of new classes of generalized nonlinear coherent states, corresponding to some, both physical and mathematical, solvable quantum system, such as half oscillator, radial part of a 3D harmonic oscillator and so on, with known discrete and (non-) degenerate spectrum. Especially, one can suggest a particularization: $\lambda = 2k - \frac{1}{2}$, where k is the Bargmann index labeling the IREP. Inserting this value in Eq. (1), we obtain the radial part of the Hamiltonian of the pseudo-harmonic oscillator. Hence, a series of results obtained in this document can be found, as particular case, in [44, 45, 46].

An example of application of this technique, together with procedures provided in [47], could include design entangled nonlinear Barut- Giradello coherent states(ENBGCSs) as superpositions of multi-particle NBGCSs. ENBGCSs are coherent states with respect to generalized $su(1, 1)$ generators, and multi-particle parity states arise as a special case. Also, based on our calculations, the entangled negative binomial states and entangled squeezed states can be considered as kind of ENBGCSs. Quantum discord as well as degrees of entanglement are calculated, which are obviously depends on quantum number m and complex variable z .

- *Finally, we conclude that maximally entanglement will be achieved when the squeezing in both of field quadratures tend to zero. In other word, squeezing may be a good candidate as one of maximally entanglement witnesses.*

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